Chettinad
College of Engineering & Technology
Approved by AICTE-New Delhi and Affiliated to Anna University-Chennai

Academic Year 2025-26

Notes of Lesson

Year/Semester: II/III **Date:**	**Department : ECE** **Subject Code/Title : EC3354/Signals and Systems** **Faculty Name : Mrs.A.Karthikeyani**	**Unit : III** **Total Hours : 12** **Subject Credit : 4**

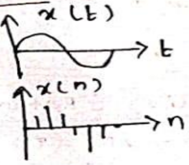
Important Points and Formulas to be Remembered

①

Signals and systems

Important points to be rememberd

1. Continuous Time (CT) signals
2. Discrete Time (DT) signals



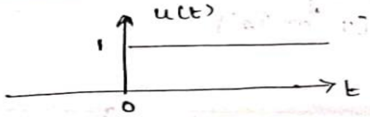
3. CT → DT [$x(t) \pm x(nT)$] $t = nT$

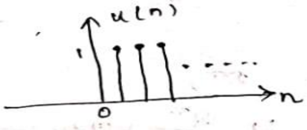
4. Elementary signals:

1. Unit step signal: ($u(t)$, $u(n)$)

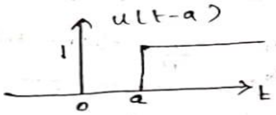
$$\boxed{\begin{aligned} u(t) &= 1, \; t \geq 0 \\ &= 0, \; t < 0 \end{aligned}}$$ [0 to ∞] [−∞ limit]

$$\boxed{\begin{aligned} u(n) &= 1, \; n \geq 0 \\ &= 0, \; n < 0 \end{aligned}}$$





If shifted, $u(t-a) = 1, \; t \geq a$ (a to ∞)
$= 0, \; t < a$



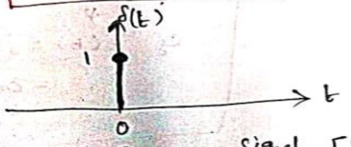
2. Unit Impulse signal [$\delta(t)$, $\delta(n)$]

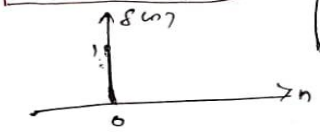
$$\boxed{\begin{aligned} \delta(t) &= +1, \; t = 0 \\ &= 0; \; t \neq 0 \end{aligned}}$$

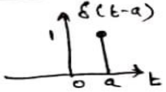
$$\boxed{\begin{aligned} \delta(n) &= 1, \; n = 0 \\ &= 0, \; n \neq 0 \end{aligned}}$$

Shifted Impulse

$$\boxed{\begin{aligned} \delta(t-a) &= 1, \; t = a \\ &= 0, \; t \neq a \end{aligned}}$$



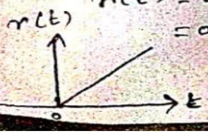


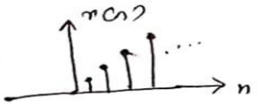


3. Unit Ramp Signal [$r(t)$, $r(n)$]

$r(t) = t, \; t \geq 0$
$= 0, \; t < 0$

$r(n) = n, \; n \geq 0$
$= 0, \; n < 0$ ⇒





Classification of Signals:

1. CT and DT ; 2. Deterministic and Non-Deterministic

3. Periodic and Aperiodic Signals:

$\boxed{\text{Periodic [Rational]}} \rightarrow$ Eg: 1.7373....

$\boxed{x(t) = x(t+T)}$ CT

$\boxed{x(n) = x(n+N)}$ DT

} Aperiodic, x(t)

[Irrational] → Eg: 1.785432....

For solving problems in CT,

$$T = \frac{2\pi}{\Omega_0} \quad [\because \pi \text{ present} \Rightarrow \text{Aperiodic}]$$

If 2 more Ω_0 values means,

$$T = \frac{T_1}{T_2} = \frac{m}{n} \Rightarrow \boxed{T = nT_1 = mT_2}$$

⇓

Substitute any T values [T_1 or T_2]

In DT,

$$N = \frac{2\pi}{\omega_0}(m)$$

↳ Min. Integer (∵ 1, 2, 3, 4, ... }
(= ω_0)

If 2 more ω_0 values means,

$$N = \frac{N_1}{N_2} = \frac{m}{n} \Rightarrow N = nN_1 = mN_2$$

Some shortcuts:

check ω_0 (or) Ω_0 value is having π multiples ⇒ said to be "Aperiodic"

⇓

except complex exponential (periodic)

⇓

$$\text{Eg}: x(t) = je^{j10t}$$

⇓

$$= \underset{T_1}{\cos 10t} + \underset{T_2}{j\sin 10t}$$

(4) Even and odd signals:

Condition:

$$\left.\begin{array}{l} x(t) = x(-t) \\ x(n) = x(-n) \end{array}\right\} \text{Even}, \quad \left.\begin{array}{l} x(t) \neq x(-t) \\ x(n) \neq x(-n) \end{array}\right\} \text{odd}.$$

Even component:

$$\boxed{\begin{array}{l} x_e(t) = \frac{1}{2}(x(t) + x(-t)) \\ x_e(n) = 1/2\,(x(n) + x(-n)) \end{array}}$$

Odd component:

$$\boxed{\begin{array}{l} x_o(t) = 1/2\,(x(t) - x(-t)) \\ x_o(n) = 1/2\,(x(n) - x(-n)) \end{array}}$$

(5) Energy and Power signals:

[E] [P]

Condition: $\boxed{E = \text{Finite} \; ; P = 0}$ ⇒ "Energy signal"

$\boxed{E = \infty \; ; P = \text{Finite}}$ ⇒ "Power signal".

Formula: "CT"

$$\boxed{\begin{array}{l} E = \lim\limits_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt \text{ Joules.} \\ P = \lim\limits_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \text{ Watts.} \end{array}}$$

"DT":

$$\boxed{\begin{array}{l} E = \sum\limits_{n=-\infty}^{\infty} |x(n)|^2 \\ P = \lim\limits_{N \to \infty} \frac{1}{2N+1} \sum\limits_{n=-N}^{N} |x(n)|^2 \end{array}}$$

R.M.S value = $\sqrt{P}$ ⊛

(6). Causal and NonCausal signal:

||

Present and past (x(n-2))

(future I/P)

Eg: x(n+2)

Classification of systems:

(1) Static and dynamic Systems:

$\Downarrow$ (Static) $\qquad\qquad$ $\Downarrow$ (dynamic)

Eg: $2x(n)$, $\qquad$ Memory oriended.

$x(n)\cdot y(n)$ $\qquad$ $\frac{d^2y(t)}{dt^2}$, $y(n-1)$, ...

$\Downarrow$

"Memoryless

(2) Linear and Non linear system:

$$T[ax_1(t)+bx_2(t)] = aT[x_1(t)]+bT[x_2(t)]$$

(*) Shortcut: if the expression is having denominator, square term, log terms → said to be Non-linear.

For L.H.S, Multiply throughout with a and b.

For R.H.S, Replace, $x(t)$ as $ax_1(t)+bx_2(t)$

$y(t)$ as $ay_1(t)+by_2(t)$

(3) Time Variant system:

(*)
$$\left.\boxed{\begin{aligned} y(t,T) &= y(t-T) \\ y(n,N) &= y(n-N) \end{aligned}}\right] \text{Time Invariant (TIV)}$$

If not equal 'TV' system.

& $y(t,T)$ means, in function substitute $t-T$ instead of t.

$y(t-T)$ " whenever the 't' comes ~~replace~~ as write '$-T$' along with function:

Eg: $y(t,T) = t\,x(t-T)$ $\qquad$ | $\qquad$ $y(n,N) = x(-(n-N))$

$y(t-T) = (t-T)\,x(t-T)$ $\qquad$ | $\qquad$ $y(n-N) = x(-n+N)$

(4) Causal System & Non-causal system:

$\Downarrow$

$y(t)$ depends on present & past values of $x(t)$

Eg: $y(t) = x(t-1)$

Non-causal: Depends on future value

$y(n) = x(n+1)$

ⓑ Stable and Unstable Systems : (BIBO)

$$\int_{-\infty}^{\infty} |h(t)| < \infty \quad (OR) \quad \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Bounded input ·
Bounded output ·

'CT'

Sketch (OR) plot the signals :

For sketching the signal, substitute from starting 'n' value to ending 'n' value.

$$\text{Eg: } x(n) = \{\overset{-2}{1}, \overset{-1}{2}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{3}, \overset{2}{+1}\}$$

AB' If $x(\underbrace{-n-1}) = ?$

$-n-1 = -2 \Rightarrow -n = -1 \Rightarrow \boxed{n = 1}$ [-2 position magnitude will be plotted in n=1 position]

Likewise plot from -2 to 2

If $x(-n/2) = ?$

$-n/2 = -2 \Rightarrow -n = -4 \Rightarrow \boxed{n = 4} \Rightarrow$ Here -2 position value will be plotted in n=4 position.

UNIT - II

(1) Fourier Series :

(a) Trigonometric Fourier Series :

ⓧ $$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\Omega_0 t) + b_n \sin(n\Omega_0 t)]$$

$$a_0 = 1/T \int_{t_0}^{t_0+T} x(t)\, dt$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos(n\Omega_0 t)\, dt$$

$$b_n = 2/T \int_{t_0}^{t_0+T} x(t) \sin(n\Omega_0 t)\, dt$$

(b) Exponential Fourier series :

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega_0 t} \; ; \; C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jn\Omega_0 t}\, dt$$

(c) Cosine representation:

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\Omega_0 t + \theta_n)$$

$$A_0 = a_0 ; \quad A_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Dirichlet's condition (or) Existence of Fourier series.

(1) ~~Has~~ x(t) → Have only a finite no. of maxima & minima.

(2) " → Have a finite no. of discontinuities.

(3) " → Absolutely Integrable over one period, $\int_0^T |x(t)|\,dt < \infty$

Fourier Transform:

$$\boxed{\begin{aligned} X(j\Omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt \\ x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega \end{aligned}}$$

"Fourier Transform pair"

Laplace Transform.

$$\boxed{\begin{aligned} X(s) &= \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt \\ x(t) &= \frac{1}{2\pi j} \int_{-\infty}^{\infty} X(s)\, e^{st}\, ds \end{aligned}}$$

"Laplace Transform pair"

UNIT - III . [Linear Time Invariant - Continuous Time systems [LTI - CT]

Differential Equation - solution:

$(X(j\Omega), X(s))$

I/P ⇒ $x(t)$; O/P ⇒ ($y(t)$) response; Impulse Response ⇒ ($h(t)$)

Transfer function ⇒ $H(s)$

If step Response, sub. $x(t) = u(t)$ and find $y(t)$.

If Impulse Response, find $h(t)$

Steps to solve:

(1) Take Laplace Transform on both side,

With Initial condition	Without initial condition.
$\frac{d^3y(t)}{dt^3} = s^3Y(s) - s^2y(0) - sy'(0) - y''(0)$	$\frac{d^3y(t)}{dt^3} = s^3Y(s)$
$\frac{d^2y(t)}{dt^2} = s^2Y(s) - sy(0) - y'(0)$	$\frac{d^2y(t)}{dt^2} = s^2Y(s)$
$\frac{dy(t)}{dt} = sY(s) - y(0)$	$\frac{dy(t)}{dt} = sY(s)$
$y(t) = Y(s)$	$y(t) = Y(s)$

Ilk for $x(t)$ also:

(2) Find $H(s) = \frac{Y(s)}{X(s)}$ ⇒ Apply partial fraction .. after finding Roots..

(3) Take inverse Laplace Transform for $H(s) \Rightarrow h(t)$.

(2) Block diagram: (Realization). $[\,\frac{1}{s}\,]$

1. Direct form - I $Y(s) =$ → ② $W(s) =$ → ①
2. Direct form - II ⇒ $H(s) = \frac{Y(s)}{X(s)}$
3. Cascade form ⇒ $H(s) = H_1(s) \cdot H_2(s) \cdot H_3(s) \ldots$
4. parallel form ⇒ $H(s) = H_1(s) + H_2(s) + H_3(s) + \ldots$

UNIT - IV

① Discrete Time Fourier Transform $[X(e^{j\omega})]$

$$\boxed{\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \\ x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega \end{aligned}}$$

DTFT pair.

② z - Transform:

$$\boxed{\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \\ x(n) &= \frac{1}{2\pi j}\oint_{C} X(z)\, z^{n-1}\, dz \end{aligned}}$$

$\Rightarrow$ z - Transform pair

For Inverse z-Transform:

(1) power series expansion (long division)

↳ (a) causal $\quad$ (b) Non-causal $\Rightarrow \{a, \ldots a, b\}$ $\rightarrow z, z^2, \ldots$

↓ (Answer $= \{\underset{\uparrow}{a}, b, c, d, \ldots\}$

Divided result will have $z^{-1}, z^{-2} \ldots$

(2) partial fraction method.

Unit - V

Linear Time Invariant - Discrete Time System (LTI-DT)

Convolution sum:

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$Z\{x(n) * h(n)\} = X(z)\, H(z)$$

Convolution → Linear → Matrix, Tabular, graphical
(N = L + M − 1)
Convolution → Circular → Concentric circle, Matrix Multiplication

N → y(n) seq. length
L → x(n) " "
M → h(n) " "

Note: In circular, x(n) & h(n) no. of datas should be equal.

Difference Equation:

$$H(z) = \frac{Y(z)}{X(z)}$$

$$h(n) = z^{-1}[H(z)]$$

Without Initial Condition,	With Initial condition
$Z\{y(n-3)\} = z^{-3}Y(z)$	$Z\{y(n-1)\} = z^{-1}Y(z) + y(-1)$
$Z\{y(n-2)\} = z^{-2}Y(z)$	$Z\{y(n-2)\} = z^{-2}Y(z) + z^{-1}y(-1) + y(-2)$
$Z\{y(n-1)\} = z^{-1}Y(z)$	$Z\{y(n-3)\} = z^{-3}Y(z) + z^{-2}y(-1) + z^{-1}y(-2) + y(-3)$
$Z\{y(n)\} = Y(z)$	

Block diagr:

Direct form, I, II & cascade form is same with Unit - III

Find Roots & Apply partial fraction

Parallel form ⇒ $H(z) = C + H_1(z) + H_2(z) + \dots$

⇓

Through Long division (Anti causal)